



तपसो मा ज्ञानिर्गम्य
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VIDYAPEETH ACADEMY

IIT JEE | NEET | FOUNDATION

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JEE Main 2023 (Memory based)

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Answer & Solutions

MATHEMATICS

1. Two lines are given as $\frac{x-2}{3} = \frac{y-1}{3} = \frac{z-0}{2}$ and $\frac{x-1}{3} = \frac{y-2}{2} = \frac{z-1}{3}$ Then the shortest distance between the lines is:

- A. $\frac{6}{\sqrt{43}}$
- B. $\frac{11}{\sqrt{43}}$
- C. $\frac{3}{\sqrt{43}}$
- D. $\frac{5}{\sqrt{43}}$

Answer (B)

Sol.

$$\text{Shortest distance} = \frac{|\vec{a}_2 - \vec{a}_1 \cdot \vec{b}_1 \times \vec{b}_2|}{|\vec{b}_1 \times \vec{b}_2|}$$

From given data

$$\vec{a}_2 - \vec{a}_1 = (2-1)\hat{i} + (1-2)\hat{j} + (0-1)\hat{k} = \hat{i} - \hat{j} - \hat{k}$$

$$\vec{b}_1 = 3\hat{i} + 3\hat{j} + 2\hat{k}$$

$$\vec{b}_2 = 3\hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 3 & 2 \\ 3 & 2 & 3 \end{vmatrix} = 5\hat{i} - 3\hat{j} - 3\hat{k}$$

$$\text{Shortest Distance} = \frac{11}{\sqrt{43}}$$

2. Tangent is drawn at a point on the parabola $y^2 = 24x$. It intersects the hyperbola $xy = 2$ at points A and B . Locus of mid point of AB is:

- A. $y^2 = 3x$
- B. $y^2 = -3x$
- C. $y^2 = 6x$
- D. $y^2 = -6x$

Answer (B)

$$\frac{\begin{vmatrix} 1 & -1 & -1 \\ 3 & 3 & 2 \\ 3 & 2 & 3 \end{vmatrix}}{|5\hat{i} - 3\hat{j} - 3\hat{k}|}$$

Sol.

Let a point on $y^2 = 24x$ be $(6t^2, 12t)$

Equation of tangent $\equiv 12yt = 12(x + 6t^2)$

$$\Rightarrow ty = x + 6t^2 \quad \dots (1)$$

Let midpoint of chord AB be (h, k)

Equation of chord bisect at this point is:

$$\frac{xk + hy}{2} = hk \quad \dots (2)$$

Comparing (1) and (2) we get,

$$\frac{t}{h/2} = \frac{-1}{k/2} = \frac{6t^2}{hk}$$

$$\Rightarrow t = \frac{-h}{k} \text{ and } -2h = 6\left(\frac{-h}{k}\right)^2$$

$$\Rightarrow -2hk^2 = 6h^2$$

$$\Rightarrow k^2 = -3h$$

The required locus is $y^2 = -3x$

3. $\lim_{t \rightarrow 0} [(1^{\frac{1}{\sin^2 t}} + 2^{\frac{1}{\sin^2 t}} + 3^{\frac{1}{\sin^2 t}} + \dots + n^{\frac{1}{\sin^2 t}})^{\sin^2 t}]$

- A. 0
- B. n
- C. $\frac{n^2-n}{2}$
- D. $n^2 + n$

Answer (B)

Sol.

$$\lim_{t \rightarrow 0} n \left[\left(\frac{1}{n}\right)^{\csc^2 t} + \left(\frac{2}{n}\right)^{\csc^2 t} + \left(\frac{3}{n}\right)^{\csc^2 t} + \dots + \left(\frac{n}{n}\right)^{\csc^2 t} \right]^{\sin^2 t}$$

$$\Rightarrow n[0 + 0 + \dots + 1]$$

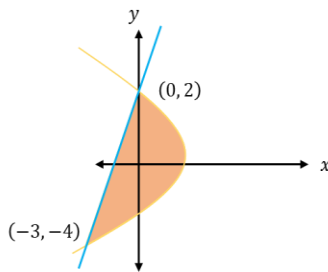
$$\Rightarrow n \times 1 = n$$

4. The area enclosed between $y^2 = -4x + 4$ and $y = 2x + 2$ is:

- A. 3
- B. 6
- C. 9
- D. 12

Answer (C)

Sol.



$$\begin{aligned} \text{Required area} &= \int_{-4}^2 \left(\frac{4-y^2}{4} - \frac{y-2}{2} \right) dy \\ &= \left[2y - \frac{y^3}{12} - \frac{y^2}{4} \right]_{-4}^2 \\ &= \left(4 - \frac{8}{12} - 1 \right) - \left(-8 + \frac{16}{3} - 4 \right) \\ &= \left(3 - \frac{2}{3} \right) + 12 - \frac{16}{3} \\ &= 9 \end{aligned}$$

5. $\sum_{r=0}^{22} {}^{22}C_r {}^{23}C_r$ is equal to:

- A. ${}^{44}C_{22}$
- B. ${}^{45}C_{23}$
- C. ${}^{45}C_{24}$
- D. ${}^{44}C_{23}$

Answer (B)

Sol.

$$\begin{aligned} \sum_{r=0}^{22} {}^{22}C_r {}^{23}C_r &= \sum_{r=0}^{22} {}^{22}C_r {}^{23}C_{23-r} \\ &= {}^{22}C_0 {}^{23}C_{23} + {}^{22}C_1 {}^{23}C_{22} + \dots + {}^{22}C_{21} {}^{23}C_2 + {}^{22}C_{22} {}^{23}C_1 \\ (1+x)^{22} &= {}^{22}C_0 + {}^{22}C_1 x + \dots + {}^{22}C_{21} x^{21} + {}^{22}C_{22} x^{22} \\ (1+x)^{23} &= {}^{23}C_0 + {}^{23}C_1 x + \dots + {}^{23}C_{22} x^{22} + {}^{23}C_{23} x^{23} \\ \text{coefficient of } x^{23} \text{ in } (1+x)^{22}(1+x)^{23} &= \sum_{r=0}^{22} {}^{22}C_r {}^{23}C_{23-r} \\ &= {}^{45}C_{23} \end{aligned}$$

6. $\sim(\sim p \wedge q) \Rightarrow (\sim p \vee q)$ is equivalent to:

- A. $\sim p \vee q$
- B. $\sim p \wedge q$
- C. $p \wedge q$
- D. $p \vee q$

Answer (A)

Sol.

$$\begin{aligned} \sim(\sim p \wedge q) &\Rightarrow (\sim p \vee q) & [p \rightarrow q \Leftrightarrow \sim p \vee q] \\ &= (\sim p \wedge q) \vee (\sim p \vee q) \\ &= (\sim p \vee (\sim p \wedge q)) \wedge (q \vee (\sim p \vee q)) \\ &= (\sim p \vee q) \wedge (q \vee \sim p) \\ &= \sim p \vee q \end{aligned}$$

7. There are 12 languages. One can choose at most 2 from 5 particular languages. The number of ways in which one can select 5 languages is:

- A. 540
- B. 535
- C. 546
- D. 525

Answer (C)

Sol.

Case -1: If no language is selected from given 5 particular languages

$$\Rightarrow {}^7C_5$$

Case -2: If 1 language is chosen from the given 5 and 4 from other 7 languages

$$\Rightarrow {}^7C_4 {}^5C_1$$

Case -3: If 2 languages are chosen from the given 5 and 3 from other 7 languages

$$\begin{aligned} &\Rightarrow {}^7C_3 {}^5C_2 \\ \therefore \text{Total ways} &= {}^7C_5 + {}^7C_4 {}^5C_1 + {}^7C_3 {}^5C_2 \\ &= 21 + 175 + 350 \\ &= 546 \end{aligned}$$

8. The solution of differential equation $\frac{dy}{dx} + \frac{y}{x^2} = \frac{1}{x^3}$ is:

- A. $y = \left(1 + \frac{1}{x}\right) + ce^{\frac{1}{x}}$
- B. $y = \left(1 - \frac{1}{x}\right) + ce^{\frac{1}{x}}$
- C. $y = \left(x + \frac{1}{x}\right) + ce^{\frac{1}{x}}$
- D. $y = \left(x - \frac{1}{x}\right) + ce^{\frac{1}{x}}$

Answer (A)

Sol.

Given equation is linear differential equation

$$\begin{aligned} \therefore I.F &= e^{\int \frac{1}{x^2} dx} = e^{-\left(\frac{1}{x}\right)} \\ \Rightarrow \int d(ye^{-\frac{1}{x}}) &= \int \frac{e^{-\frac{1}{x}}}{x^3} dx \\ \Rightarrow ye^{-\frac{1}{x}} &= \frac{1}{x} e^{-\frac{1}{x}} + e^{-\frac{1}{x}} + c \\ \Rightarrow y &= \left(\frac{1}{x} + 1\right) + ce^{\frac{1}{x}} \end{aligned}$$

9. The equation $x^2 - [x] + x + 3 = x[x]^2$ has (where $[.]$ represents greatest integer function)

- A. No solution
- B. 1 solution in $(-\infty, 1)$
- C. 2 solution in $(-\infty, \infty)$
- D. 1 solution in $(-\infty, \infty)$

Answer (D)

Sol.

$$\begin{aligned} x^2 - [x] + x + 3 &= x[x]^2 \\ x^2 + \{x\} + 3 &= x[x]^2 \end{aligned}$$

Case 1: $x < 0$

$$\text{L.H.S} > 0, \text{R.H.S} < 0$$

\therefore No solution

Case 2: $x \in [0, 1)$

$$\text{L.H.S} \geq 3, \text{R.H.S} = 0$$

No solution

Case 3: $x \in [1, 2)$

$$\text{L.H.S} \in [4, 8), \text{R.H.S} \in [1, 2)$$

No solution

Case 4: $x \in [2, 3)$

$$\text{L.H.S} \in [7, 13), \text{R.H.S} \in [8, 12)$$

$$\therefore x^2 + x - 2 + 3 = 4x$$

$$\Rightarrow x^2 - 3x + 1 = 0$$

$$\Rightarrow x = \frac{3+\sqrt{5}}{2} \text{ one solution}$$

Case 5: $x \in [3,4)$

L.H.S $\in [12,20)$, R.H.S $\in [27,36)$

Similarly, For $x > 4$, R.H.S $>$ L.H.S Always

\therefore one solution in $[1,2)$ or only one solution in $(-\infty, \infty)$

10. The function $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$ is:

- A. Continuous but non-differentiable at $x = 0$
- B. Discontinuous at $x = 0$
- C. $f'(x)$ is differentiable but not continuous
- D. $f'(x)$ is continuous but non-differentiable

Answer (D)

Sol.

$$f(x) = x^2 \sin\left(\frac{1}{x}\right)$$

At $x = 0$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^-} f(x) = f(0)$$

$f(x)$ is continuous at $x = 0$

$$\text{LHD at } x = 0 \text{ is } \lim_{h \rightarrow 0} \frac{f(0-h) - f(0)}{-h}$$

$$\lim_{h \rightarrow 0} \frac{-h^2 \sin\left(\frac{1}{h}\right)}{-h} = 0$$

$$\text{RHD at } x = 0 \text{ is } \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} = 0$$

$\therefore f(x)$ is continuous as well as differentiable hence Option A and B are wrong.

A function cannot be differentiable unless continuous hence Option C is wrong.

11. The distance of the point $P(1,2,3)$ from the plane containing the points $A(1, 4, 5)$, $B(2, 3, 4)$ and $C(3, 2, 1)$ is equal to:

- A. $5\sqrt{2}$
- B. $3\sqrt{2}$
- C. 1
- D. $\sqrt{2}$

Answer (D)

Sol.

Direction ratio of $\overrightarrow{AB} = \langle 1, -1, -1 \rangle$

Direction ratio of $\overrightarrow{AC} = \langle 2, -2, -4 \rangle$

Vector normal to \overrightarrow{AB} and $\overrightarrow{AC} = (1,1,0)$

Equation of plane $(x-1, y-4, z-5) \cdot (1, 1, 0) = 0$
 $\Rightarrow x + y - 5 = 0$

Distance from $(1,2,3) = \left| \frac{1+2-5}{\sqrt{1^2+1^2}} \right| = \sqrt{2}$

12. If $(1 + \sqrt{3}i)^{200} = 2^{199}(p + iq)$, then $p + q + q^2$ and $p - q + q^2$ are roots of the equation:

- A. $x^2 - 4x + 1 = 0$
- B. $x^2 - 4x - 1 = 0$
- C. $x^2 + 4x + 1 = 0$
- D. $x^2 + 4x - 1 = 0$

Answer (A)

Sol.

$$\begin{aligned}
 (1 + \sqrt{3}i)^{200} &= 2^{199}(p + iq) \\
 \Rightarrow 2^{200} \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i \right)^{200} &= 2^{199}(p + iq) \\
 \Rightarrow 2^{200} \left(\cos \left(\frac{\pi}{3} \right) + i \sin \left(\frac{\pi}{3} \right) \right)^{200} &= 2^{199}(p + iq) \\
 \Rightarrow 2^{200} \left(e^{i\frac{\pi}{3}} \right)^{200} &= 2^{199}(p + iq) \\
 \Rightarrow 2^{200} \left(\cos \left(\frac{200\pi}{3} \right) + i \sin \left(\frac{200\pi}{3} \right) \right) &= 2^{199}(p + iq) \\
 \Rightarrow 2^{200} \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i \right) &= 2^{199}(p + iq) \\
 \Rightarrow 2^{199}(-1 + \sqrt{3}i) &= 2^{199}(p + iq) \\
 \Rightarrow p = -1, q = \sqrt{3} \\
 p + q + q^2 &= 2 + \sqrt{3} \\
 p - q + q^2 &= 2 - \sqrt{3} \\
 \text{So, the equation whose roots are } 2 + \sqrt{3} \text{ and } 2 - \sqrt{3} \text{ is} \\
 x^2 - (2 + \sqrt{3} + (2 - \sqrt{3}))x + (2 + \sqrt{3})(2 - \sqrt{3}) &= 0 \\
 \Rightarrow x^2 - 4x + 1 &= 0
 \end{aligned}$$

13. $\sum_{r=0}^{2023} r^2 \cdot {}^{2023}C_r = \alpha \cdot 2023 \cdot 2^{2022}$ then α is equal to:

- A. 1012
- B. 1011
- C. 1020
- D. 1022

Answer (A)

Sol.

$$\begin{aligned}
 \sum_{r=0}^{2023} r^2 \cdot {}^{2023}C_r &= \sum_{r=0}^{2023} (r^2 - r) \cdot {}^{2023}C_r + \sum_{r=0}^{2023} r \cdot {}^{2023}C_r \\
 &= 2023 \cdot 2022 \cdot \sum_{r=2}^{2023} {}^{2021}C_{r-2} + 2023 \sum_{r=1}^{2023} {}^{2022}C_{r-1} \\
 &= 2023 \cdot 2022 \cdot 2^{2021} + 2023 \cdot 2^{2022} \\
 &= 2^{2022} \cdot 2023 \cdot 1012
 \end{aligned}$$

Then $\alpha = 1012$

14. If $y^2 + \ln(\cos^2 x) = y$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ then

- A. $y''(0) = 0$
- B. $|y''(0)| = 2$
- C. $y'(0) = 3$
- D. $y'(0) = -3$

Answer (B)

Sol.

Differentiating both sides we have

$$2yy' + \frac{1}{\cos^2 x} \cdot (2 \cos x) \cdot (-\sin x) = y'$$

$$2yy' - 2 \tan x = y' \dots (i)$$

Differentiating both sides again we have

$$\Rightarrow 2(y')^2 - 2yy'' - 2 \sec^2 x = y'' \dots (ii)$$

From (i) substituting $x = 0$

$$y'(0) = 0$$

and substituting $x = 0$ and $y'(0) = 0$ in (ii)

$$y''(0) = -2$$

$$|y''(0)| = 2$$

15. if $\vec{v} \cdot \vec{w} = 2$, $\vec{u} \times \vec{w} = \vec{v} + \alpha \vec{u}$, $\vec{u} = 2\hat{i} + 3\hat{j} - 4\hat{k}$, $\vec{v} = \hat{i} + 2\hat{j} - 4\hat{k}$ then $\vec{u} \cdot \vec{w}$

- A. $\frac{28}{12}$
- B. $\frac{12}{29}$
- C. $-\frac{29}{12}$
- D. $\frac{29}{12}$

Answer (D)

Sol.

$$\text{If } \vec{u} \times \vec{w} = \vec{v} + \alpha \vec{u}$$

$$\vec{v} \cdot \vec{w} + \alpha \vec{u} \cdot \vec{w} = \vec{0} \quad (\text{taking dot product with } \vec{v})$$

$$\alpha(\vec{u} \cdot \vec{w}) = -2 \dots (i)$$

$$0 = \vec{u} \cdot \vec{v} + \alpha |\vec{u}|^2$$

$$\Rightarrow 29\alpha + 24 = 0$$

$$\Rightarrow \alpha = -\frac{24}{29}$$

From (i)

$$\vec{u} \cdot \vec{w} = \frac{-2}{\left(-\frac{24}{29}\right)} = \frac{58}{24} = \frac{29}{12}$$

16. If $R = \{(a, b) : g.c.d(a, b) = 1, a, b \in Z\}$. Then relation R is:

- A. Reflexive
- B. Symmetric
- C. Transitive
- D. None of the above

Answer (B)

Sol.

Reflexive relation:-

$$(a, a) \in R \forall a \in A$$

$$\text{Let } a = 5, g.c.d(5,5) = 5 \neq 1$$

$\Rightarrow R$ is not reflexive relation

Symmetric Relation:-

$$\text{If } (a, b) \in R \Rightarrow (b, a) \in R$$

$$\text{If } g.c.d(a, b) = 1 \Rightarrow g.c.d(b, a) = 1$$

Transitive Relation:-

$$\text{If } (a, b) \text{ and } (c, c) \in R \Rightarrow (a, c) \in R$$

$$(2,3) \text{ and } (3,4) \in R \text{ but } (2,4) \notin R \quad (\text{because } g.c.d(2,4) = 2)$$

$\therefore R$ is symmetric relation

17. The sum of all the values of x satisfying $\cos^{-1} x - 2 \sin^{-1} x = \cos^{-1} 2x$ is:

- A. 0
- B. 1
- C. $\frac{1}{2}$
- D. $-\frac{1}{2}$

Answer (A)

Sol.

$$\cos^{-1} x - 2 \sin^{-1} x = \cos^{-1} 2x$$

$$\Rightarrow \frac{\pi}{2} - 3 \sin^{-1} x = \cos^{-1} 2x$$

$$\Rightarrow \cos \left(\frac{\pi}{2} - 3 \sin^{-1} x \right) = \cos (\cos^{-1} 2x)$$

$$\Rightarrow \sin (3 \sin^{-1} x) = \cos (\cos^{-1} 2x)$$

$$\Rightarrow 3x - 4x^3 = 2x$$

$$\Rightarrow 4x^3 = x$$

$$\Rightarrow x = 0, \pm \frac{1}{2}$$

Sum of values of $x = 0$

18. The value of $12 \int_0^3 |x^2 - 3x + 2| dx$ is equal to:

Answer (22)

Sol.

$$\begin{aligned} I &= \int_0^3 |x^2 - 3x + 2| dx = \int_0^3 |(x-1)(x-2)| dx = \int_0^1 (x-1)(x-2) dx + \int_1^2 (x-1)(x-2) dx + \int_2^3 (x-1)(x-2) dx \\ &= \int_0^1 (x-1)(x-2) dx - \int_1^2 (x-1)(x-2) dx + \int_2^3 (x-1)(x-2) dx \\ &= \int_0^1 (x-1)(x-2) dx - \int_1^2 (x-1)(x-2) dx + \int_2^3 (x-1)(x-2) dx \\ &= \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x \right]_0^1 - \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x \right]_1^2 + \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x \right]_2^3 \\ &= \left(\frac{1}{3} - \frac{3}{2} + 2 \right) - \left\{ \left(\frac{8}{3} - 6 + 4 \right) - \left(\frac{1}{3} - \frac{3}{2} + 2 \right) \right\} + \left\{ \left(9 - \frac{27}{2} + 6 \right) - \left(\frac{8}{3} - 6 + 4 \right) \right\} \\ &= \frac{11}{6} \end{aligned}$$

$$\text{Hence } 12 \int_0^3 |x^2 - 3x + 2| dx = \int_0^3 |x^2 - 3x + 2| dx = 12I = 12 \times \frac{11}{6} = 22$$