FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Wednesday 06th SEPTEMBER, 2020) TIME : 3 PM to 6 PM

MATHEMATICS — TEST PAPER WITH SOLUTION

1. The set of all real values of λ for which the function $f(x) = (1 - \cos^2 x).(\lambda + \sin x)$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, has exactly one maxima and exactly one minima, is :

(1) $\left(-\frac{1}{2}, \frac{1}{2}\right) - \{0\}$ (2) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

(3) $\left(-\frac{3}{2}, \frac{3}{2}\right)$ (4) $\left(-\frac{3}{2}, \frac{3}{2}\right) - \{0\}$

Official Ans. by NTA (4)

Sol. $f(x) = (1 - \cos^2 x)(\lambda + \sin x)$

$x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$f(x) = \lambda \sin^2 x + \sin^3 x$

$f'(x) = 2\lambda \sin x \cos x + 3\sin^2 x \cos x$

$f'(x) = \sin x \cos x(2\lambda + 3\sin x)$

$\sin x = 0, \frac{-2\lambda}{3}$, $(\lambda \neq 0)$

for exactly one maxima & minima

$\frac{-2\lambda}{3} \in (-1, 1) \Rightarrow \lambda \in \left(\frac{-3}{2}, \frac{3}{2}\right)$

$\lambda \in \left(-\frac{3}{2}, \frac{3}{2}\right) - \{0\}$

2. For all twice differentiable functions $f : R \to R$, with $f(0) = f(1) = f'(0) = 0$

(1) $f''(x) = 0$, for some $x \in (0, 1)$

(2) $f''(0) = 0$

(3) $f''(x) \neq 0$ at every point $x \in (0, 1)$

(4) $f''(x) = 0$ at every point $x \in (0, 1)$

Official Ans. by NTA (1)

Sol. $f(0) = f(1) = f'(0) = 0$

Apply Rolles theorem on $y = f(x)$ in $x \in [0, 1]$

$f(0) = f(1) = 0$

$\Rightarrow f'(\alpha) = 0$ where $\alpha \in (0, 1)$

Now apply Rolles theorem on $y = f'(x)$

in $x \in [0, \alpha]$

$f'(0) = f'(\alpha) = 0$ and $f'(x)$ is continuous and differentiable

$\Rightarrow f''(\beta) = 0$ for some , $\beta \in (0, \alpha) \in (0, 1)$

$\Rightarrow f''(x) = 0$ for some $x \in (0, 1)$

3. If the tangent to the curve, $y = f(x) = x\log_e x$, $(x > 0)$ at a point $(c, f(c))$ is parallel to the line - segement joining the points (1, 0) and (e, e), then c is equal to :

(1) $\frac{1}{e-1}$ (2) $e^{\left(\frac{1}{1-e}\right)}$

(3) $e^{\left(\frac{1}{e-1}\right)}$ (4) $\frac{e-1}{e}$

Official Ans. by NTA (3)

Sol. $f(x) = x\log_e x$

$$f'(x)\Big|_{(c,f(c))} = \frac{e-0}{e-1}$$

$f'(x) = 1 + \log_e x$

$$f'(x)\Big|_{(c,f(c))} = 1 + \log_e c = \frac{e}{e-1}$$

$$\log_e c = \frac{e-(e-1)}{e-1} = \frac{1}{e-1} \Rightarrow c = e^{\frac{1}{e-1}}$$

4. Consider the statement : "For an integer n, if $n^3 - 1$ is even, then n is odd." The contrapositive statement of this statement is :

(1) For an integer n, if $n^3 - 1$ is not even, then n is not odd.

(2) For an integer n, if n is even, then $n^3 - 1$ is odd.

(3) For an integer n, if n is odd, then $n^3 - 1$ is even.

(4) For an integer n, if n is even, then $n^3 - 1$ is even.

Official Ans. by NTA (2)

Sol. Contrapositive of $(p \rightarrow q)$ is $\sim q \rightarrow \sim p$

For an integer n, if n is even then $(n^3 - 1)$ is odd

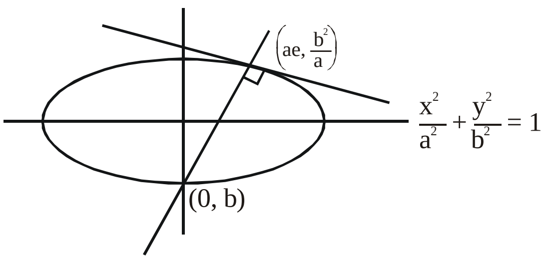
5. If the normal at an end of a latus rectum of an ellipse passes through an extremity of the minor axis, then the eccentricity e of the ellipse satisfies :

(1) $e^2 + 2e - 1 = 0$ (2) $e^2 + e - 1 = 0$

(3) $e^4 + 2e^2 - 1 = 0$ (4) $e^4 + e^2 - 1 = 0$

Official Ans. by NTA (4)

Sol.



$$\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2e^2$$

$$\frac{a^2x}{ae} - \frac{b^2y}{b^2}.a = a^2e^2$$

$$\frac{ax}{e} - ay = a^2e^2 \Rightarrow \frac{x}{e} - y = ae^2$$

passes through (0, b)

$-b = ae^2 \Rightarrow b^2 = a^2e^4$

$a^2(1 - e^2) = a^2e^4 \Rightarrow e^4 + e^2 = 1$

6. A plane P meets the coordinate axes at A, B and C respectively. The centroid of ΔABC is given to be (1, 1, 2). Then the equation of the line through this centroid and perpendicular to the plane P is :

(1) $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$

(2) $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$

(3) $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-2}{1}$

(4) $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-2}{2}$

Official Ans. by NTA (2)

Sol. $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

$A \equiv (a, 0, 0)$, $B \equiv (0, b, 0)$, $C \equiv (0, 0, c)$

Centroid $\equiv \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right) = (1, 1, 2)$

$a = 3$, $b = 3$, $c = 6$

Plane : $\frac{x}{3} + \frac{y}{3} + \frac{z}{6} = 1$

$2x + 2y + z = 6$

line $\perp$ to the plane (DR of line = $2\hat{i} + 2\hat{j} + \hat{k}$)

$$\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$$

7. If α and β are the roots of the equation $2x(2x + 1) = 1$, then β is equal to :

(1) $2\alpha^2$ (2) $2\alpha(\alpha + 1)$

(3) $-2\alpha(\alpha + 1)$ (4) $2\alpha(\alpha - 1)$

Official Ans. by NTA (3)

Sol. α and β are the roots of the equation $4x^2 + 2x - 1 = 0$

$4\alpha^2 + 2\alpha = 1 \Rightarrow \frac{1}{2} = 2\alpha^2 + \alpha$ ...(1)

$\beta = \frac{-1}{2} - \alpha$

using equation (1)

$\beta = -(2\alpha^2 + \alpha) - \alpha$

$\beta = -2\alpha^2 - 2\alpha$

$\beta = -2\alpha(\alpha + 1)$

8. Let $z = x + iy$ be a non-zero complex number such that $z^2 = i|z|^2$, where $i = \sqrt{-1}$, then z lies on the :

(1) imaginary axis (2) real axis

(3) line, $y = x$ (4) line, $y = -x$

Official Ans. by NTA (3)

Sol. $z = x + iy$

$z^2 = i|z|^2$

$(x + iy)^2 = i(x^2 + y^2)$

$(x^2 - y^2) - i(x^2 + y^2 - 2xy) = 0$

$(x - y)(x + y) - i(x - y)^2 = 0$

$(x - y)((x + y) - i(x - y)) = 0$

$\Rightarrow x = y$

z lies on $y = x$

9. The common difference of the A.P. $b_1, b_2, ..., b_m$ is 2 more than the common difference of A.P. $a_1, a_2, ..., a_n$. If $a_{40} = -159$, $a_{100} = -399$ and $b_{100} = a_{70}$, then b_1 is equal to :

(1) –127 (2) –81

(3) 81 (4) 127

Official Ans. by NTA (2)

Sol. $a_1, a_2, ..., a_n \to (CD = d)$

$b_1, b_2, ..., b_m \to (CD = d + 2)$

$a_{40} = a + 39d = -159$ ...(1)

$a_{100} = a + 99d = -399$ ...(2)

Subtract : $60d = -240 \Rightarrow d = -4$

using equation (1)

$a + 39(-4) = -159$

$a = 156 - 159 = -3$

$a_{70} = a + 69d = -3 + 69(-4) = -279 = b_{100}$

$b_{100} = -279$

$b_1 + 99(d + 2) = -279$

$b_1 - 198 = -279 \Rightarrow b_1 = -81$

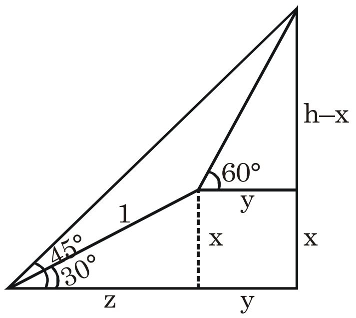
10. The angle of elevation of the summit of a mountain from a point on the ground is 45°. After climding up one km towards the summit at an inclination of 30° from the ground, the angle of elevation of the summit is found to be 60°. Then the height (in km) of the summit from the ground is :

(1) $\frac{1}{\sqrt{3}-1}$ (2) $\frac{1}{\sqrt{3}+1}$

(3) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ (4) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

Official Ans. by NTA (1)

Sol.



$\sin 30° = x \Rightarrow x = \frac{1}{2}$

$\cos 30° = z \Rightarrow z = \frac{\sqrt{3}}{2}$

$\tan 45° = \frac{h}{y+z} \Rightarrow h = y + z$

$\tan 60° = \dfrac{h-x}{y} \Rightarrow \tan 60° = \dfrac{h-x}{h-z}$

$\sqrt{3}(h-z) = h-x$

$(\sqrt{3}-1)h = \sqrt{3}z - x$

$\Rightarrow (\sqrt{3}-1)h = \dfrac{3}{2} - \dfrac{1}{2}$

$\Rightarrow (\sqrt{3}-1)h = 1$

$h = \dfrac{1}{\sqrt{3}-1}$

11. Let $\theta = \dfrac{\pi}{5}$ and $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$. If B = A + A^4, then det(B) :

(1) is one (2) lies in (1, 2)

(3) is zero (4) lies in (2, 3)

Official Ans. by NTA (2)

Sol. $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

B = A + A^4

$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$

$B = \begin{bmatrix} (\cos\theta + \cos 4\theta) & (\sin\theta + \sin 4\theta) \\ -(\sin\theta + \sin 4\theta) & (\cos\theta + \cos 4\theta) \end{bmatrix}$

$|B| = (\cos\theta + \cos 4\theta)^2 + (\sin\theta + \sin 4\theta)^2$

$|B| = 2 + 2\cos 3\theta$, when $\theta = \dfrac{\pi}{5}$

$|B| = 2 + 2\cos\dfrac{3\pi}{5} = 2(1 - \sin 18)$

$|B| = 2\left(1 - \dfrac{\sqrt{5}-1}{4}\right) = 2\left(\dfrac{5-\sqrt{5}}{4}\right) = \dfrac{5-\sqrt{5}}{2}$

12. For a suitably chosen real constant a, let a function, $f : R - \{-a\} \to R$ be defined by $f(x) = \dfrac{a-x}{a+x}$. Further suppose that for any real number $x \neq -a$ and $f(x) \neq -a$, $(f o f)(x) = x$. Then $f\left(-\dfrac{1}{2}\right)$ is equal to :

(1) $\dfrac{1}{3}$ (2) 3

(3) –3 (4) $-\dfrac{1}{3}$

Official Ans. by NTA (2)

Sol. $f(x) = \dfrac{a-x}{a+x}$ $x \in R - \{-a\} \to R$

$f(f(x)) = \dfrac{a - f(x)}{a + f(x)} = \dfrac{a - \left(\dfrac{a-x}{a+x}\right)}{a + \left(\dfrac{a-x}{a+x}\right)}$

$f(f(x)) = \dfrac{(a^2 - a) + x(a+1)}{(a^2 + a) + x(a-1)} = x$

$\Rightarrow (a^2 - a) + x(a + 1) = (a^2 + a)x + x^2(a - 1)$

$\Rightarrow a(a - 1) + x(1 - a^2) - x^2(a - 1) = 0$

$\Rightarrow a = 1$

$f(x) = \dfrac{1-x}{1+x}$,

$f\left(\dfrac{-1}{2}\right) = \dfrac{1 + \frac{1}{2}}{1 - \frac{1}{2}} = 3$

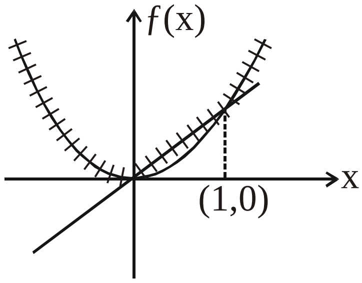
13. Let $f : R \to R$ be a function defined by $f(x) = \max\{x, x^2\}$. Let S denote the set of all points in R, where f is not differentiable. Then :

(1) {0, 1} (2) {0}

(3) ϕ(an empty set) (4) {1}

Official Ans. by NTA (1)

Sol. $f(x) = \max(x, x^2)$



Non-differentiable at x = 0, 1

S = {0, 1}

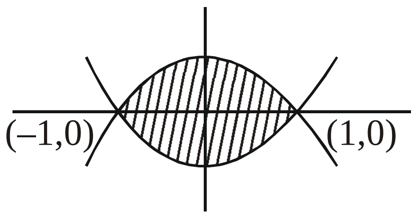
14. The area (in sq. units) of the region enclosed by the curves $y = x^2 - 1$ and $y = 1 - x^2$ is equal to :

(1) $\frac{4}{3}$ (2) $\frac{8}{3}$

(3) $\frac{16}{3}$ (4) $\frac{7}{2}$

Official Ans. by NTA (2)

Sol. $y = x^2 - 1$ and $y = 1 - x^2$



$$A = \int_{-1}^{1}\left((1-x^2)-(x^2-1)\right)dx$$

$$A = \int_{-1}^{1}(2-2x^2)dx = 4\int_{0}^{1}(1-x^2)dx$$

$$A = 4\left(x-\frac{x^3}{3}\right)_0^1 = 4\left(\frac{2}{3}\right) = \frac{8}{3}$$

15. The probabilities of three events A, B and C are given by P(A) = 0.6, P(B) = 0.4 and P(C) = 0.5. If $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$, $P(A \cap B \cap C) = 0.2$, $P(B \cap C) = \beta$ and $P(A \cup B \cup C) = \alpha$, where $0.85 \le \alpha \le 0.95$, then β lies in the interval:

(1) [0.36, 0.40] (2) [0.35, 0.36]

(3) [0.25, 0.35] (4) [0.20, 0.25]

Official Ans. by NTA (3)

Sol. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.8 = 0.6 + 0.4 - P(A \cap B)$

$P(A \cap B) = 0.2$

$P(A \cup B \cup C) = \Sigma P(A) - \Sigma P(A \cap B) + P(A \cap B \cap C)$

$\alpha = 1.5 - (0.2 + 0.3 + \beta) + 0.2$

$\alpha = 1.2 - \beta \in [0.85, 0.95]$

(where $\alpha \in [0.85, 0.95]$)

$\beta \in [0.25, 0.35]$

16. if the constant term in the binomial expansion of $\left(\sqrt{x}-\frac{k}{x^2}\right)^{10}$ is 405, then |k| equals :

(1) 2 (2) 1

(3) 3 (4) 9

Official Ans. by NTA (3)

Sol. $\left(\sqrt{x}-\frac{k}{x^2}\right)^{10}$

$$T_{r+1} = {}^{10}C_r\left(\sqrt{x}\right)^{10-r}\left(\frac{-k}{x^2}\right)^r$$

$$T_{r+1} = {}^{10}C_r.x^{\frac{10-r}{2}}.(-k)^r.x^{-2r}$$

$$T_{r+1} = {}^{10}C_r x^{\frac{10-5r}{2}}(-k)^r$$

Constant term : $\frac{10-5r}{2} = 0 \Rightarrow r = 2$

$T_3 = {}^{10}C_2.(-k)^2 = 405$

$$k^2 = \frac{405}{45} = 9$$

$k = \pm 3 \Rightarrow |k| = 3$

17. The integral $\int_1^2 e^x.x^x\left(2+\log_e x\right)dx$ equal :

(1) e(4e + 1) (2) e(2e – 1)

(3) $4e^2 - 1$ (4) e(4e – 1)

Official Ans. by NTA (4)

Sol. $\int_1^2 e^x.x^x\left(2+\log_e x\right)dx$

$$\int_1^2 e^x\left(2x^x+x^x\log_e x\right)dx$$

$$\int_1^2 e^x\left(\underbrace{x^x}_{f(x)}+\underbrace{x^x\left(1+\log_e x\right)}_{f'(x)}\right)dx$$

$$\left(e^x.x^x\right)_1^2=4e^2-e$$

18. Let L denote the line in the xy-plane with x and y intercepts as 3 and 1 respectively. Then the image of the point (–1, –4) in this line is :

(1) $\left(\frac{8}{5},\frac{29}{5}\right)$ (2) $\left(\frac{29}{5},\frac{11}{5}\right)$

(3) $\left(\frac{11}{5},\frac{28}{5}\right)$ (4) $\left(\frac{29}{5},\frac{8}{5}\right)$

Official Ans. by NTA (3)

Sol. L : $\frac{x}{3}+\frac{y}{1}=1 \Rightarrow x + 3y - 3 = 0$

Image of point (–1, –4)

$$\frac{x+1}{1}=\frac{y+4}{3}=-2\left(\frac{-1-12-3}{10}\right)$$

$$\frac{x+1}{1}=\frac{y+4}{3}=\frac{16}{5}$$

$$(x,y)\equiv\left(\frac{11}{5},\frac{28}{5}\right)$$

19. If $y=\left(\frac{2}{\pi}x-1\right)\operatorname{cosec}x$ is the solution of the differential equation,

$\frac{dy}{dx}+p(x)y=\frac{2}{\pi}\operatorname{cosec}x,\ 0<x<\frac{\pi}{2}$, then the function p(x) is equal to

(1) cotx (2) tanx

(3) cosecx (4) secx

Official Ans. by NTA (1)

Sol. $y=\left(\frac{2x}{\pi}-1\right)\operatorname{cosec}x$ ...(1)

$$\frac{dy}{dx}=\frac{2}{\pi}\operatorname{cosec}x-\left(\frac{2x}{\pi}-1\right)\operatorname{cosec}x\cot x$$

$$\frac{dy}{dx}=\frac{2\operatorname{cosec}x}{\pi}-y\cot x$$

using equation (1)

$$\frac{dy}{dx}+y\cot x=\frac{2\operatorname{cosec}x}{\pi}$$

$$\frac{dy}{dx}+p(x).y=\frac{2\operatorname{cosec}x}{\pi}\quad x\in\left(0,\frac{\pi}{2}\right)$$

Compare : p(x) = cot x

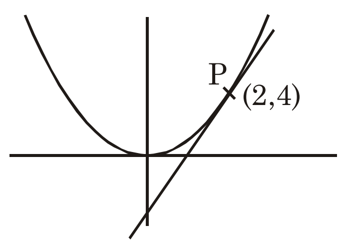
20. The centre of the circle passing through the point (0, 1) and touching the parabola $y = x^2$ at the point (2, 4) is :

(1) $\left(\frac{3}{10},\frac{16}{5}\right)$ (2) $\left(\frac{-16}{5},\frac{53}{10}\right)$

(3) $\left(\frac{6}{5},\frac{53}{10}\right)$ (4) $\left(\frac{-53}{10},\frac{16}{5}\right)$

Official Ans. by NTA (2)

Sol.



$y = x^2$

$$\left.\frac{dy}{dx}\right|_P=4$$

$(y-4)=4(x-2)$

$4x - y - 4 = 0$

Circle : $(x - 2)^2 + (y - 4)^2 + \lambda(4x - y - 4) = 0$

passes through (0, 1)

$4 + 9 + \lambda(-5) = 0 \Rightarrow \lambda = \dfrac{13}{5}$

Circle : $x^2 + y^2 + x(4\lambda - 4) + y(-\lambda - 8) + (20 - 4\lambda) = 0$

Centre : $\left(2 - 2\lambda, \dfrac{\lambda + 8}{2}\right) \equiv \left(\dfrac{-16}{5}, \dfrac{53}{10}\right)$

21. The sum of distinct values of λ for which the system of equations

$(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$

$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$

$2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0,$

has non-zero solutions, is ___________.

Official Ans. by NTA (3.00)

Sol. $(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$

$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$

$2x + (3\lambda + 1)y + (3\lambda - 3)z = 0$

$$\begin{vmatrix} \lambda - 1 & 3\lambda + 1 & 2\lambda \\ \lambda - 1 & 4\lambda - 2 & \lambda + 3 \\ 2 & 3\lambda + 1 & 3\lambda - 3 \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$$\begin{vmatrix} 0 & 3 - \lambda & \lambda - 3 \\ \lambda - 3 & \lambda - 3 & -2(\lambda - 3) \\ 2 & 3\lambda + 1 & 3\lambda - 3 \end{vmatrix} = 0$$

$$(\lambda - 3)^2 \begin{vmatrix} 0 & -1 & 1 \\ 1 & 1 & -2 \\ 2 & 3\lambda + 1 & 3\lambda - 3 \end{vmatrix} = 0$$

$(\lambda - 3)^2 [(3\lambda + 1) + (3\lambda - 1)] = 0$

$6\lambda(\lambda - 3)^2 = 0 \Rightarrow \lambda = 0, 3$

Sum = 3

22. Suppose that a function $f : R \to R$ satisfies $f(x + y) = f(x)f(y)$ for all $x, y \in R$ and $f(1) = 3$. If $\sum_{i=1}^{n} f(i) = 363$, then n is equal to __________.

Official Ans. by NTA (5.00)

Sol. $f(x + y) = f(x)\,f(y)$

put $x = y = 1$ $\quad f(2) = (f(1))^2 = 3^2$

put $x = 2, y = 1$ $\quad f(3) = (f(1))^3 = 3^3$

$\vdots$

Similarly $f(x) = 3^x$

$\sum_{i=1}^{n} f(i) = 363 \Rightarrow \sum_{i=1}^{n} 3^i = 363$

$(3 + 3^2 + \ldots + 3^n) = 363$

$\dfrac{3(3^n - 1)}{2} = 363$

$3^n - 1 = 242 \Rightarrow 3^n = 243$

$\Rightarrow n = 5$

23. The number of words (with or without meaning) that can be formed from all the letters of the word "LETTER" in which vowels never come together is ___________.

Official Ans. by NTA (120.00)

Sol. LETTER

vowels = EE, consonant = LTTR

_ L _ T _ T _ R _

$\dfrac{4!}{2!} \times {}^5C_2 \times \dfrac{2!}{2!} = 12 \times 10 = 120$

24. Consider the data on x taking the values 0, 2, 4, 8, ..., 2^n with frequencies ${}^nC_0$, ${}^nC_1$, ${}^nC_2$, ..., ${}^nC_n$ respectively. If the mean of this data is $\frac{728}{2^n}$, then n is equal to ____________.

Official Ans. by NTA (6.00)

Sol.

x	0	2	4	8		2^n
f	${}^nC_0$	${}^nC_1$	${}^nC_2$	${}^nC_3$		${}^nC_n$

$$\text{Mean} = \frac{\sum x_i f_i}{\sum f_i} = \frac{\sum_{r=1}^{n} 2^r \, {}^nC_r}{\sum_{r=0}^{n} {}^nC_r}$$

$$\text{Mean} = \frac{(1+2)^n - {}^nC_0}{2^n} = \frac{728}{2^n}$$

$$\Rightarrow \frac{3^n - 1}{2^n} = \frac{728}{2^n}$$

$\Rightarrow 3^n = 729 \Rightarrow n = 6$

25. If $\vec{x}$ and $\vec{y}$ be two non-zero vectors such that $|\vec{x}+\vec{y}| = |\vec{x}|$ and $2\vec{x}+\lambda\vec{y}$ is perpendicular to $\vec{y}$, then the value of λ is __________.

Official Ans. by NTA (1.00)

Sol. $|\vec{x}+\vec{y}| = |\vec{x}|$

$$\sqrt{|\vec{x}|^2 + |\vec{y}|^2 + 2\vec{x}.\vec{y}} = |\vec{x}|$$

$$|\vec{y}|^2 + 2\vec{x}.\vec{y} = 0 \qquad .... (1)$$

Now $(2\vec{x}+\lambda\vec{y}).\vec{y} = 0$

$$2\vec{x}.\vec{y} + \lambda|\vec{y}|^2 = 0$$

from (1)

$$-|\vec{y}|^2 + \lambda|\vec{y}|^2 = 0$$

$$(\lambda - 1)|\vec{y}|^2 = 0$$

given $|\vec{y}| \neq 0 \qquad \Rightarrow \lambda = 1$