FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Thursday 03rd SEPTEMBER, 2020) TIME : 9 AM to 12 PM

MATHEMATICS — TEST PAPER WITH SOLUTION

1. A die is thrown two times and the sum of the scores appearing on the die is observed to be a multiple of 4. Then the conditional probability that the score 4 has appeared atleast once is :

(1) $\frac{1}{8}$ (2) $\frac{1}{9}$

(3) $\frac{1}{3}$ (4) $\frac{1}{4}$

Official Ans. by NTA (2)

Sol. A : Sum obtained is a multiple of 4.

A = {(1, 3), (2, 2), (3, 1), (2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (6, 6)}

B : Score of 4 has appeared at least once.

B = {(1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4), (4, 1), (4, 2), (4, 3), (4, 5), (4, 6)}

Required probability = $P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$

$$= \frac{1/36}{9/36} = \frac{1}{9}$$

2. The lines

$\vec{r} = (\hat{i} - \hat{j}) + \ell(2\hat{i} + \hat{k})$ and

$\vec{r} = (2\hat{i} - \hat{j}) + m(\hat{i} + \hat{j} - \hat{k})$

(1) Intersect when ℓ = 1 and m = 2

(2) Intersect when ℓ = 2 and m = $\frac{1}{2}$

(3) Do not intersect for any values of ℓ and m

(4) Intersect for all values of ℓ and m

Official Ans. by NTA (3)

Sol. $\vec{r} = \hat{i}(1 + 2\ell) + \hat{j}(-1) + \hat{k}(\ell)$

$\vec{r} = \hat{i}(2 + m) + \hat{j}(m - 1) + \hat{k}(-m)$

For intersection

$1 + 2\ell = 2 + m$ ...... (i)

$-1 = m - 1$ ...... (ii)

$\ell = -m$ ...... (iii)

from (ii) m = 0

from (iii) ℓ = 0

These values of m and ℓ do not satisfy equation (1).

Hence the two lines do not intersect for any values of ℓ and m.

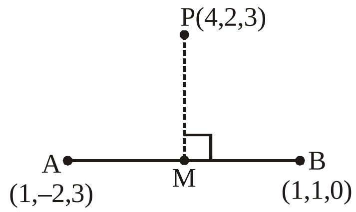
3. The foot of the perpendicular drawn from the point (4, 2, 3) to the line joining the points (1, –2, 3) and (1, 1, 0) lies on the plane :

(1) x + 2y – z = 1 (2) x – 2y + z = 1

(3) x – y – 2z = 1 (4) 2x + y – z = 1

Official Ans. by NTA (4)

Sol.



Equation of AB = $\vec{r} = (\hat{i} + \hat{j}) + \lambda(3\hat{j} - 3\hat{k})$

Let coordinates of M = (1, (1 + 3λ), –3λ).

$\overrightarrow{PM} = -3\hat{i} + (3\lambda - 1)\hat{j} - 3(\lambda + 1)\hat{k}$

$\overrightarrow{AB} = 3\hat{j} - 3\hat{k}$

$\because$ $\overrightarrow{PM} \perp \overrightarrow{AB} \Rightarrow \overrightarrow{PM} \cdot \overrightarrow{AB} = 0$

$\Rightarrow$ 3(3λ – 1) + 9(λ + 1) = 0

$\Rightarrow \lambda = -\frac{1}{3}$

$\therefore$ M = (1, 0, 1)

Clearly M lies on 2x + y – z = 1.

4. A hyperbola having the transverse axis of length $\sqrt{2}$ has the same foci as that of the ellipse $3x^2 + 4y^2 = 12$, then this hyperbola does not pass through which of the following points ?

(1) $\left(1, -\frac{1}{\sqrt{2}}\right)$ (2) $\left(\sqrt{\frac{3}{2}}, \frac{1}{\sqrt{2}}\right)$

(3) $\left(\frac{1}{\sqrt{2}}, 0\right)$ (4) $\left(-\sqrt{\frac{3}{2}}, 1\right)$

Official Ans. by NTA (2)

Sol. Ellipse : $\frac{x^2}{4} + \frac{y^2}{3} = 1$

eccentricity = $\sqrt{1 - \frac{3}{4}} = \frac{1}{2}$

$\therefore$ foci = $(\pm 1, 0)$

for hyperbola, given $2a = \sqrt{2} \Rightarrow a = \frac{1}{\sqrt{2}}$

$\therefore$ hyperbola will be

$$\frac{x^2}{1/2} - \frac{y^2}{b^2} = 1$$

eccentricity = $\sqrt{1+2b^2}$

$\therefore$ foci = $\left(\pm\sqrt{\frac{1+2b^2}{2}}, 0\right)$

$\because$ Ellipse and hyperbola have same foci

$$\Rightarrow \sqrt{\frac{1+2b^2}{2}} = 1$$

$$\Rightarrow b^2 = \frac{1}{2}$$

$\therefore$ Equation of hyperbola : $\frac{x^2}{1/2} - \frac{y^2}{1/2} = 1$

$$\Rightarrow x^2 - y^2 = \frac{1}{2}$$

Clearly $\left(\sqrt{\frac{3}{2}}, \frac{1}{\sqrt{2}}\right)$ does not lie on it.

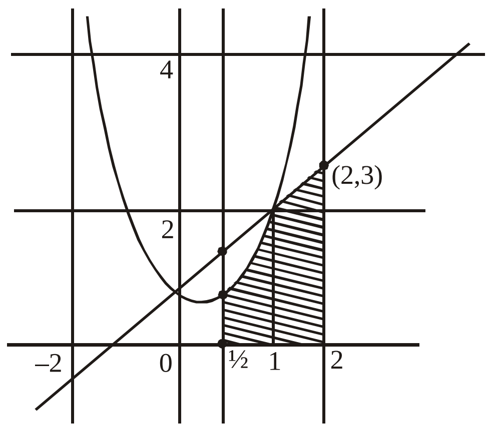
5. The area (in sq. units) of the region $\{(x, y) : 0 \le y \le x^2 + 1, 0 \le y \le x + 1, \frac{1}{2} \le x \le 2\}$ is :

(1) $\frac{79}{16}$ (2) $\frac{23}{6}$

(3) $\frac{79}{24}$ (4) $\frac{23}{16}$

Official Ans. by NTA (3)

Sol. $0 \le y \le x^2 + 1, 0 \le y \le x + 1, \frac{1}{2} \le x \le 2$



Required area = $\int_{1/2}^{1} (x^2+1)dx + \frac{1}{2}(2+3)\times 1$

$$= \frac{19}{24} + \frac{5}{2} = \frac{79}{24}$$

6. If the first term of an A.P. is 3 and the sum of its first 25 terms is equal to the sum of its next 15 terms, then the common difference of this A.P. is :

(1) $\frac{1}{4}$ (2) $\frac{1}{5}$

(3) $\frac{1}{7}$ (4) $\frac{1}{6}$

Official Ans. by NTA (4)

Sol. Sum of 1st 25 terms = sum of its next 15 terms

$\Rightarrow (T_1 + ... + T_{25}) = (T_{26} + ... + T_{40})$

$\Rightarrow (T_1 + ... + T_{40}) = 2(T_1 + ... + T_{25})$

$\Rightarrow \frac{40}{2}[2 \times 3 + (39d)] = 2 \times \frac{25}{2}[2 \times 2 + 24d]$

$\Rightarrow d = \frac{1}{6}$

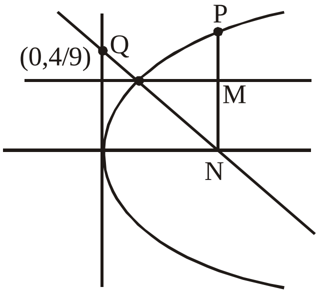
7. Let P be a point on the parabola, $y^2 = 12x$ and N be the foot of the perpendicular drawn from P on the axis of the parabola. A line is now drawn through the mid-point M of PN, parallel to its axis which meets the parabola at Q. If the y-intercept of the line NQ is $\frac{4}{3}$, then :

(1) MQ = $\frac{1}{3}$ (2) PN = 3

(3) MQ = $\frac{1}{4}$ (4) PN = 4

Official Ans. by NTA (3)

Sol.



Let $P = (3t^2, 6t)$; $N = (3t^2, 0)$

$M = (3t^2, 3t)$

Equation of MQ : $y = 3t$

$\therefore Q = \left(\frac{3}{4}t^2, 3t\right)$

Equation of NQ

$$y = \frac{3t}{\left(\frac{3}{4}t^2 - 3t^2\right)}(x - 3t^2)$$

y-intercept of NQ $= 4t = \frac{4}{3} \Rightarrow t = \frac{1}{3}$

$\therefore MQ = \frac{9}{4}t^2 = \frac{1}{4}$

$PN = 6t = 2$

8. For the frequency distribution :

Variate (x) :	x_1	x_2	x_3 .....x_{15}
Frequency (f) :	f_1	f_2	f_3 ....f_{15}

where $0 < x_1 < x_2 < x_3 < .... < x_{15} = 10$ and $\sum_{i=1}^{15} f_i > 0$, the standard deviation cannot be :

(1) 2 (2) 1

(3) 4 (4) 6

Official Ans. by NTA (4)

Sol. $\because \sigma^2 \le \frac{1}{4}(M - m)^2$

Where M and m are upper and lower bounds of values of any random variable.

$\therefore \sigma^2 < \frac{1}{4}(10 - 0)^2$

$\Rightarrow 0 < \sigma < 5$

$\therefore \sigma \neq 6.$

9. $\int_{-\pi}^{\pi} \left| \pi - |x| \right| dx$ is equal to :

(1) π^2 (2) $2\pi^2$

(3) $\sqrt{2}\pi^2$ (4) $\frac{\pi^2}{2}$

Official Ans. by NTA (1)

Sol. $\int_{-\pi}^{\pi} \left|\pi - |x|\right| dx = 2\int_{0}^{\pi} |\pi - x| dx$

$$= 2\int_{0}^{\pi} (\pi - x) dx$$

$$= 2\left[\pi x - \frac{x^2}{2}\right]_0^{\pi} = \pi^2$$

10. Consider the two sets :

A = {m ∈ R : both the roots of $x^2 - (m + 1)x + m + 4 = 0$ are real} and B = [–3, 5).

Which of the following is not true ?

(1) A – B = (–∞, –3) ∪ (5, ∞)

(2) A ∩ B = {–3}

(3) B – A = (–3, 5)

(4) A ∪ B = R

Official Ans. by NTA (1)

Sol. A : D ≥ 0

$\Rightarrow (m + 1)^2 - 4(m + 4) \geq 0$

$\Rightarrow m^2 + 2m + 1 - 4m - 16 \geq 0$

$\Rightarrow m^2 - 2m - 15 \geq 0$

$\Rightarrow (m - 5)(m + 3) \geq 0$

$\Rightarrow m \in (-\infty, -3] \cup [5, \infty)$

$\therefore$ A = (–∞, –3] ∪ [5, ∞)

B = [–3, 5)

A – B = (–∞, –3) ∪ [5, ∞)

A ∩ B = {–3}

B – A = (–3, 5)

A ∪ B = R

11. If $y^2 + \log_e(\cos^2 x) = y$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then :

(1) $|y''(0)| = 2$ (2) $|y'(0)| + |y''(0)| = 3$

(3) $|y'(0)| + |y''(0)| = 1$ (4) $y''(0) = 0$

Official Ans. by NTA (1)

Sol. $y^2 + \ln(\cos^2 x) = y$ $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

for x = 0 y = 0 or 1

Differentiating wrt x

$\Rightarrow 2yy' - 2\tan x = y'$

At (0, 0) y' = 0

At (0, 1) y' = 0

Differentiating wrt x

$2yy'' + 2(y')^2 - 2\sec^2 x = y''$

At (0, 0) y'' = –2

At (0, 1) y'' = 2

$\therefore$ |y''(0)| = 2

12. The function, $f(x) = (3x - 7)x^{2/3}$, x ∈ R, is increasing for all x lying in :

(1) $(-\infty, 0) \cup \left(\frac{3}{7}, \infty\right)$

(2) $(-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$

(3) $\left(-\infty, \frac{14}{15}\right)$

(4) $\left(-\infty, -\frac{14}{15}\right) \cup (0, \infty)$

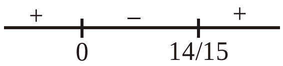
Official Ans. by NTA (2)

Sol. $f(x) = (3x - 7)x^{2/3}$

$\Rightarrow f(x) = 3x^{5/3} - 7x^{2/3}$

$\Rightarrow f'(x) = 5x^{2/3} - \dfrac{14}{3x^{1/3}}$

$$= \frac{15x - 14}{3x^{1/3}} > 0$$



$\therefore f'(x) > 0 \ \forall \ x \in (-\infty, 0) \cup \left(\dfrac{14}{15}, \infty\right)$

13. The value of $(2.{}^1P_0 - 3.{}^2P_1 + 4.{}^3P_2 - \ldots$ up to 51^{th} term) + $(1! - 2! + 3! - \ldots$ up to 51^{th} term) is equal to :

(1) $1 + (51)!$ (2) $1 - 51(51)!$

(3) $1 + (52)!$ (4) 1

Official Ans. by NTA (3)

Sol. $S = (2.{}^1p_0 - 3.{}^2p_1 + 4.{}^3p_2 \ldots\ldots\ldots$ upto 51 terms)

$+ (1! + 2! + 3! \ldots\ldots\ldots\ldots$ upto 51 terms)

$[\because {}^np_{n-1} = n!]$

$\therefore S = (2 \times 1! - 3 \times 2! + 4 \times 3! \ldots. + 52.51!)$

$+ (1! - 2! + 3! \ldots\ldots\ldots\ldots (51)!)$

$= (2! - 3! + 4! \ldots\ldots\ldots + 52!)$

$+ (1! - 2! + 3! - 4! + \ldots\ldots + (51)!)$

$= 1! + 52!.$

14. If $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} =$

$Ax^3 + Bx^2 + Cx + D$, then $B + C$ is equal to :

(1) -1 (2) 1

(3) -3 (4) 9

Official Ans. by NTA (3)

Sol. $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix}$

$= Ax^3 + Bx^2 + Cx + D.$

$R_2 \to R_2 - R_1 \qquad R_3 \to R_3 - R_2$

$\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-1 & x-1 & x-1 \\ x-2 & 2(x-2) & 6(x-2) \end{vmatrix}$

$= (x - 1)(x - 2) \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 1 & 1 & 1 \\ 1 & 2 & 6 \end{vmatrix}$

$= -3(x - 1)^2 (x - 2) = -3x^3 + 12x^2 - 15x + 6$

$\therefore B + C = 12 - 15 = -3$

15. The solution curve of the differential equation, $(1 + e^{-x})(1 + y^2)\dfrac{dy}{dx} = y^2$, which passes through the point (0, 1), is :

(1) $y^2 = 1 + y \log_e\left(\dfrac{1+e^x}{2}\right)$

(2) $y^2 + 1 = y\left(\log_e\left(\dfrac{1+e^x}{2}\right) + 2\right)$

(3) $y^2 = 1 + y \log_e\left(\dfrac{1+e^{-x}}{2}\right)$

(4) $y^2 + 1 = y\left(\log_e\left(\dfrac{1+e^{-x}}{2}\right) + 2\right)$

Official Ans. by NTA (1)

Sol. $(1 + e^{-x})(1 + y^2)\dfrac{dy}{dx} = y^2$

$\Rightarrow (1 + y^{-2})\,dy = \left(\dfrac{e^x}{1+e^x}\right)dx$

$\Rightarrow \left(y - \dfrac{1}{y}\right) = \ell n(1+e^x) + c$

$\therefore$ It passes through (0, 1) $\Rightarrow c = -\ell n 2$

$\Rightarrow y^2 = 1 + y\,\ell n\left(\dfrac{1+e^x}{2}\right)$

16. If the number of integral terms in the expansion of $(3^{1/2} + 5^{1/8})^n$ is exactly 33, then the least value of n is :

(1) 264 (2) 256

(3) 128 (4) 248

Official Ans. by NTA (2)

Sol. $T_{r+1} = {}^nC_r (3)^{\frac{n-r}{2}} (5)^{\frac{r}{8}}$ $(n \ge r)$

Clearly r should be a multiple of 8.

$\because$ there are exactly 33 integral terms

Possible values of r can be

0, 8, 16, ... , 32 × 8

$\therefore$ least value of n = 256.

17. If α and β are the roots of the equation $x^2 + px + 2 = 0$ and $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$ are the roots of the equation $2x^2 + 2qx + 1 = 0$, then $\left(\alpha - \dfrac{1}{\alpha}\right)\left(\beta - \dfrac{1}{\beta}\right)\left(\alpha + \dfrac{1}{\beta}\right)\left(\beta + \dfrac{1}{\alpha}\right)$ is equal to:

(1) $\dfrac{9}{4}(9 + p^2)$ (2) $\dfrac{9}{4}(9 - q^2)$

(3) $\dfrac{9}{4}(9 - p^2)$ (4) $\dfrac{9}{4}(9 + q^2)$

Official Ans. by NTA (3)

Sol. α, β are roots of $x^2 + px + 2 = 0$

$\Rightarrow \alpha^2 + p\alpha + 2 = 0$ & $\beta^2 + p\beta + 2 = 0$

$\Rightarrow \dfrac{1}{\alpha}, \dfrac{1}{\beta}$ are roots of $2x^2 + px + 1 = 0$

But $\dfrac{1}{\alpha}, \dfrac{1}{\beta}$ are roots of $2x^2 + 2qx + 1 = 0$

$\Rightarrow p = 2q$

Also $\alpha + \beta = -p$ $\alpha\beta = 2$

$$\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$$

$$= \left(\frac{\alpha^2 - 1}{\alpha}\right)\left(\frac{\beta^2 - 1}{\beta}\right)\left(\frac{\alpha\beta + 1}{\beta}\right)\left(\frac{\alpha\beta + 1}{\alpha}\right)$$

$$= \frac{(-p\alpha - 3)(-p\beta - 3)(\alpha\beta + 1)^2}{(\alpha\beta)^2}$$

$$= \frac{9}{4}(p\alpha\beta + 3p(\alpha + \beta) + 9)$$

$$= \frac{9}{4}(9 - p^2) = \frac{9}{4}(9 - 4q^2)$$

18. Let [t] denote the greatest integer ≤ t. If for some $\lambda \in R - \{0, 1\}$, $\lim\limits_{x\to 0}\left|\dfrac{1 - x + |x|}{\lambda - x + [x]}\right| = L$, then L is equal to :

(1) 1 (2) 2

(3) $\dfrac{1}{2}$ (4) 0

Official Ans. by NTA (2)

Sol. LHL : $\lim\limits_{x\to 0^-}\left|\dfrac{1 - x - x}{\lambda - x - 1}\right| = \left|\dfrac{1}{\lambda - 1}\right|$

RHL : $\lim\limits_{x\to 0^+}\left|\dfrac{1 - x + x}{\lambda - x + 1}\right| = \left|\dfrac{1}{\lambda}\right|$

For existence of limit

LHL = RHL

$$\Rightarrow \frac{1}{|\lambda-1|}=\frac{1}{|\lambda|}\Rightarrow \lambda=\frac{1}{2}$$

$$\therefore \quad L=\frac{1}{|\lambda|}=2$$

19. $2\pi-\left(\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{16}{65}\right)$ is equal to:

(1) $\frac{7\pi}{4}$ (2) $\frac{5\pi}{4}$

(3) $\frac{3\pi}{2}$ (4) $\frac{\pi}{2}$

Official Ans. by NTA (3)

Sol. $2\pi-\left(\sin^{-1}\left(\frac{4}{5}\right)+\sin^{-1}\left(\frac{5}{13}\right)+\sin^{-1}\left(\frac{16}{65}\right)\right)$

$$=2\pi-\left(\tan^{-1}\left(\frac{4}{3}\right)+\tan^{-1}\left(\frac{5}{12}\right)+\tan^{-1}\left(\frac{16}{63}\right)\right)$$

$$=2\pi-\left(\tan^{-1}\left(\frac{63}{16}\right)+\tan^{-1}\left(\frac{16}{63}\right)\right)$$

$$=2\pi-\frac{\pi}{2}=\frac{3\pi}{2}$$

20. The proposition $p \rightarrow \sim(p \wedge \sim q)$ is equivalent to:

(1) $(\sim p) \vee q$ (2) q

(3) $(\sim p) \wedge q$ (4) $(\sim p) \vee (\sim q)$

Official Ans. by NTA (1)

Sol. $p \rightarrow \sim(p\wedge \sim q)$

$$=\sim p \vee \sim(p\wedge \sim q)$$

$$=\sim p \vee \sim p \vee q$$

$$=\sim(p\wedge q)\vee q$$

$$=\sim p \vee q$$

21. Let $A=\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$, $x \in R$ and $A^4=[a_{ij}]$. If $a_{11}=109$, then a_{22} is equal to _______ .

Official Ans. by NTA (10)

Sol. $A=\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$

$$A^2=\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}=\begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$$

$$A^4=\begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}\begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$$

$$=\begin{bmatrix} (x^2+1)^2+x^2 & x(x^2+1)+x \\ x(x^2+1)+x & x^2+1 \end{bmatrix}$$

$a_{11}=(x^2+1)^2+x^2=109$

$\Rightarrow x=\pm 3$

$a_{22}=x^2+1=10$

22. If $\lim_{x\to 0}\left\{\frac{1}{x^8}\left(1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cos\frac{x^2}{4}\right)\right\}=2^{-k}$, then the value of k is _______ .

Official Ans. by NTA (8)

Sol. $\lim_{x\to 0}\left\{\frac{1}{x^8}\left(1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cos\frac{x^2}{4}\right)\right\}=2^{-k}$

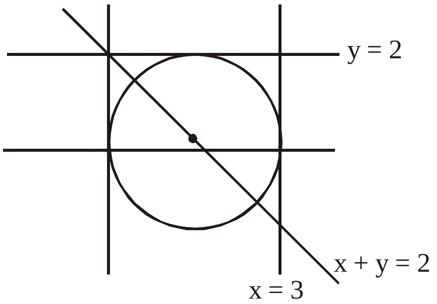
$$\Rightarrow \lim_{x\to 0}\frac{\left(1-\cos\frac{x^2}{2}\right)}{4\left(\frac{x^2}{2}\right)^2}\frac{\left(1-\cos\frac{x^2}{4}\right)}{16\left(\frac{x^2}{4}\right)^2}=\frac{1}{8}\times\frac{1}{32}=2^{-k}$$

$\Rightarrow 2^{-8}=2^{-k}\Rightarrow k=8.$

23. The diameter of the circle, whose centre lies on the line x + y = 2 in the first quadrant and which touches both the lines x = 3 and y = 2, is _______ .

Official Ans. by NTA (3)

Sol.



$\because$ center lies on x + y = 2 and in 1st quadrant

center = $(\alpha, 2 - \alpha)$

where $\alpha > 0$ and $2 - \alpha > 0 \Rightarrow 0 < \alpha < 2$

$\because$ circle touches x = 3 and y = 2

$\Rightarrow |3 - \alpha| = |2 - (2 - \alpha)| =$ radius

$\Rightarrow |3 - \alpha| = |\alpha| \Rightarrow \alpha = \frac{3}{2}$

$\therefore$ radius = α

$\Rightarrow$ Diameter = $2\alpha = 3$.

24. The value of $(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots\text{to }\infty\right)}$ is equal to _______ .

Official Ans. by NTA (4)

Sol. $(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\ldots\ldots\ldots\text{to }\infty\right)}$

$$=\left(\frac{4}{25}\right)^{\log_{\left(\frac{5}{2}\right)}\left(\frac{1}{2}\right)}$$

$$=\left(\frac{1}{2}\right)^{\log_{\left(\frac{5}{2}\right)}\left(\frac{4}{25}\right)}=\left(\frac{1}{2}\right)^{-2}=4$$

25. If $\left(\frac{1+i}{1-i}\right)^{\frac{m}{2}}=\left(\frac{1+i}{i-1}\right)^{\frac{n}{3}}=1$, $(m, n \in N)$ then the greatest common divisor of the least values of m and n is _______ .

Official Ans. by NTA (4)

Sol. $\left(\frac{1+i}{1-i}\right)^{m/2}=\left(\frac{1+i}{i-1}\right)^{n/3}=1$

$$\Rightarrow \left(\frac{(1+i)^2}{2}\right)^{m/2}=\left(\frac{(1+i)^2}{-2}\right)^{n/3}=1$$

$\Rightarrow (i)^{m/2} = (-i)^{n/3} = 1$

$\Rightarrow \frac{m}{2} = 4k_1$ and $\frac{n}{3} = 4k_2$

$\Rightarrow m = 8k_1$ and $n = 12k_2$

Least value of m = 8 and n = 12.

$\therefore$ GCD = 4